Some problems of monitoring gravitational microlensing processes

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1. GALACTIC MICROLENSING.
Related issues which need similar techniques: transients + variable stars + microlensing events.

2. MICROLENSING in EXTRAGALACTIC GLS.
- Quasar structure.
- Projected number density.
- Transversal velocities.
MICROLENSING ON THE MILKY WAY OBJECTS

Source plane

Lens plane

The deflector: microlensing mass

Model: POINT LENS, POINT SOURCE, SMALL ANGLES

Microlensing scheme

\[ y = x \left( 1 - \frac{R_E^2}{|x|^2} \right) \] -- standard

\[ y = r \left[ 1 - \left( \frac{R_0}{r} \right)^{2+\varepsilon} \right] \] -- modified
The projections onto the celestial sphere.
MOTIVATIONS FOR THE SEARCH OF THE GALACTIC GRAVITATIONAL MICROLENSING EVENTS

• Detailed investigation of the Galaxy structure in the observational field of ILMT.
• Number of faint objects.
• Exoplanets.

Observation of the Galactic microlensing light curve allows us:
• To make an independent test of the functional form of the general relativistic formula for the light deflection for impact parameters ~1 A.U.
• To test the presence of the dark matter clumps around compact objects (see Fedorova et al. MNRAS, 2016).

The necessary condition to observe Galactic microlensing events is monitoring of >10^6 sources.
Microlensing by Milky Way objects in the context of ILMT

Microlensing optical depth towards Galactic center:


Data from the ESA Gaia mission (https://www.cosmos.esa.int/gaia); Gaia Data Processing and Analysis Consortium (https://www.cosmos.esa.int/web/gaia/dpac/consortium) ESA Gaia Archive, gea.esac.esa.int (https://gea.esac.esa.int/archive/)
Distribution of GAIA stars over magnitude within the field of ILMT (ESA Gaia Archive, gea.esac.esa.int)

Total $N=3 \cdot 10^6$ stars

ILMT (expected) $N \sim 10^7$ stars
GAIA SOURCES

ILMT observation time of one source ~ 160 nights (ideal conditions)

$4^h < R.A. < 7^h$

$N = 4 \cdot 10^5$

$18^h < R.A. < 22^h$

$N = 2.5 \cdot 10^6$
Distribution of GAIA sources over distance within the field of ILMT

Total
$N = 3 \cdot 10^6$ stars

ILMT
$N \sim 10^7$ stars

(ESA Gaia Archive, gea.esac.esa.int)

Microlensing optical depth $\tau$: the probability that one star, at a specific instant in time, has an amplification caused by gravitational microlensing of $A \geq 1.34$ ($\rho$ is the mass density of microlenses, $D_s$-distance to the source).

$$\tau = \frac{2\pi GM}{3c^2} nD_s^2 = 0.9 \cdot 10^{-7} \left( \frac{\rho}{0.1M_\odot pc^{-3}} \right) \left( \frac{D_s}{3kpc} \right)^2$$
If it is possible to register $\Delta m \approx 0.01$, we must recalculate

$$R_E \rightarrow 3 R_E, \quad P \sim 9 \pi R_E^2 n$$

$$P(\Delta m > 0.01) = 0.85 \cdot 10^{-6} \left( \frac{\rho}{0.1 M_\odot pc^{-3}} \right) \left( \frac{D_s}{3 kpc} \right)^2$$

Number of events observed for $T_{obs}$ (total observation time of $N$ distant objects (160 nights) taking into account their motion

$$N_{events} = N_{sources} P(\Delta m > 0.01) \frac{T_{obs}}{T_{micro}} \sim 10 \text{yr}^{-1}$$

$$T_{micro} = \frac{2 \cdot R_E}{V_{microlens}} = 0.1 \text{yr} \left( \frac{R_E}{1 A.U.} \frac{100 \text{km/s}}{V_{microlens}} \right)$$ - effective microlensing time
NUMBER OF MICROLENSING EVENTS??

Estimated: ~10 event per year

All theory is gray, but the tree of experiment springs ever green.

The above rough estimate is not reliable because we do not know the real mass density of microlenses and their distribution. However, there are inspirational examples from history of observations of the Galactic microlensing:

- Paczyński (1991) estimated number of events with $Dm > 0.3$ mag to be $\sim 4$ per year per $10^6$ bulge stars (16 for masses $0.01 \div 0.1 \, M_{\text{sun}}$).
- Alcock et al (1997): Microlensing optical depth has appeared higher than theoretical models.
- Now OGLE group registers about thousand microlensing events per year. OGLE estimates, $\tau \sim (1\div2) \cdot 10^{-6}$.
TESTING GENERAL RELATIVITY WITH GRAVITATIONAL LENSING


MICROLENSING ON THE MILKY WAY OBJECTS

Testing the Einstein’s formula for light deflection

\[ \alpha(p) = \frac{4GM}{c^2 p} \]

The standard lens equation (lens mapping) in angular variables

\[ y = x \left( 1 - \frac{R_E^2}{|x|^2} \right) \]

The angular radius of the Einstein ring

\[ R_E = \left[ \frac{4GM D_{ds}}{c^2 D_s D_d} \right]^{1/2} \]

point lens, point source
STATEMENT OF THE PROBLEM

“Spoiled” lens mapping

\[ y = x F(\xi, \varepsilon), \]

Modified formula for light deflection

Next we consider a concrete example

\[ \alpha (p) = \left( \frac{4 GM}{c^2 p} \right)^{1+\varepsilon} \]

\[ F(\xi, \varepsilon) = 1 - \xi^{2+\varepsilon} \]

\[ \xi \equiv \frac{R_0}{r}, \quad r = |x|; \]

\[ F(\xi, 0) = 1 - \xi^2 \]

\[ R_0^{2+\varepsilon} = R_E^2 \left( \frac{4GM}{c^2 D_L} \right)^\varepsilon \]
Simulations: Estimated error of $\varepsilon$ (from a single microlensing light curve) against photometric accuracy

From one microlensing event with $\Delta m \sim 0.01 \div 0.02$ we have typically $\Delta \varepsilon \sim 0.1$.  

$$\sigma = \frac{\Delta I}{I} \approx \Delta m$$


SELECTION CRITERIA

• sufficient brightness enhancement at the maximum
• sufficient accuracy and the number of points on the microlensing light curve
• the absence of obvious signs of a binary system or planetary contribution
• absence of parallax effects
• Sample of 100 lightcurves (OGLE-2018)

\[ y = r \left[ 1 - \left( \frac{R_0}{r} \right)^{2+\varepsilon} \right] \]

Histogams of $\varepsilon$ values obtained by different methods
From $N=100$ lightcurves we have a set of $\varepsilon_j$

$$\langle \varepsilon \rangle = \sum_{j=1}^{N} w_j \varepsilon_j$$

THE RESULTS: median, mean and standard deviation obtained by three methods

<table>
<thead>
<tr>
<th>Method</th>
<th>median</th>
<th>$\langle \varepsilon \rangle$</th>
<th>$\sigma \langle \varepsilon \rangle$</th>
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</thead>
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<tr>
<td>(i)</td>
<td>0.0053</td>
<td>0.0066</td>
<td>0.0085</td>
</tr>
<tr>
<td>(ii)</td>
<td>0.0079</td>
<td>0.015</td>
<td>0.0087</td>
</tr>
<tr>
<td>(iii)</td>
<td>0.0003</td>
<td>$-0.0036$</td>
<td>0.011</td>
</tr>
</tbody>
</table>
Comparison with the accuracy of the Solar system tests Bertotti et al (Cassini mission) 2003; C.M. Will, 2014:

$$\gamma - 1 \sim 2 \cdot 10^{-5}$$

In case of Galactic microlensing: independent data of quite other sort dealing with impact distances $p \sim 10^8 \text{km}$. 
Resume:

• We tested the Einstein’s formula for the gravitational light deflection using a modified power-law dependence upon the impact distance. This involves parameter $\varepsilon$ that characterizes the deviation from the general relativistic formula (in GR $\varepsilon = 0$).

• Using one microlensing light curve we predict the error $\Delta \varepsilon \sim 0.1$ in case of the photometric accuracy $\Delta m \sim 0.01 \div 0.02$.

• Statistical treatment of 100 light curves from the sample of OGLE database on Galactic microlensing events yields $\varepsilon$ that does not contradict the General Relativity within error $\sim 0.01$.

• Involving more light curves makes the method more accurate and competitive.
Thank you for attention